

# Section 2.5: Implicit Differentiation

Objectives:

- Distinguish between functions written in implicit and explicit form.
- Use implicit differentiation to differentiate functions.

## Explicit vs. Implicit

Explicit form of an equation is one that is solved for the dependent variable. One explicit form you have come to love over the years is the slope-intercept form for a linear equation,  $y = mx + b$

Implicit form of an equation is one where the dependency of the variables is not determinable. An implicit form that you should be familiar with is the standard form for a linear equation:  $Ax + By = C$ . Two different variables are on the same side. Sometimes it is possible to isolate a variable and turn from IMPLICIT to EXPLICIT form.

### Example 1

Differentiate with respect to  $x$ .

1.  $\frac{d}{dx}x^4$

2.  $\frac{d}{dx}y^4$

3.  $\frac{d}{dx}(2x - 3y)$

4.  $\frac{d}{dx}(x^2y^3)$

<https://youtu.be/AD2-3Ze9PfQ>

## General Procedure for Implicit Differentiation

1. Differentiate with respect to  $x$ ,  $\frac{d}{dx}$
2. Collect all terms that have a  $\frac{dy}{dx}$  on one side of the equation.
3. Factor out  $\frac{dy}{dx}$  on the one side of the equation
4. Solve for  $\frac{dy}{dx}$

### Example 2

Find  $\frac{dy}{dx}$  given  $y^4 + y^2 - 3y + x^2 = -5$

<https://youtu.be/sAq7W1HD-so>

### Example 3

Find the slope of the tangent line of  $4y^2 + x^2 = 4$  at the point  $\left(\sqrt{2}, \frac{-1}{\sqrt{2}}\right)$

<https://youtu.be/3Fdx79yGtE>

### Example 4

Find the slope of the graph of  $(x^2 + y^2)^2 = \frac{100}{3}xy$  at the point  $(3,1)$ .

<https://youtu.be/20w1fbUtr7k>

Remember that a function is not differentiable at points with vertical tangents nor points at which the function is not continuous.

#### Example 5

Find  $\frac{dy}{dx}$  of the implicit equation  $\sin y = x$ . Then determine the largest interval which  $y$  is differentiable.

[https://youtu.be/bW3f\\_DU\\_VYw](https://youtu.be/bW3f_DU_VYw)

#### Example 6

Given  $x^2 + y^2 = 36$ , find  $\frac{d^2y}{dx^2}$

<https://youtu.be/SdZHVR1QjCo>

#### Example 7

Find the equation of the tangent line to  $y^2 = x^2(x^2 + y^2)$  at the point  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

<https://youtu.be/3Dn5D17wR5g>