Section 2.4: The Chain Rule

Objectives:

- Find the derivative of a composite function (including trigonometric functions) using the Chain Rule
- Derive and use the General Power Rule
- Simplify the derivatives of a function

The chain rule is one of the most powerful differentiation rules because it gives the other rules we have studied far more applicability.

Theorem: Chain Rule

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(u) = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Or

$$\frac{d}{dx} f(g(x)) = f'(g(x) \cdot g'(x)$$

Example 1

Find $\frac{dy}{dx}$ for the given function.

$f(x) = (x^3 + 1)^2$

https://youtu.be/D_WdA2hnP6k

Notice in the example above there was a similarity to the Power Rule from previous, however, instead of the power being on a variable it was on a differentiable function of *x*. We have…

Theorem: General Power Rule

Let u be a differentiable function of x and n a rational number, then

$$\frac{d}{dx}u^n = n \cdot u^{n-1} \cdot u'$$

Example 2

Find the derivative of $y = (2x + 3x^2)^3$

https://youtu.be/UIAF-QnbwY0

Example 3

For the function $f(x) = \sqrt[3]{(x^2 - 1)^2}$

Find the points on the graph where:

1. $f'(x) = 0$
2. $f'(x)$ does not exist.

https://youtu.be/XuPKNKpL23k

Example 4

Differentiate the function: $g(t) = \dfrac{-5}{(2t - 3)^2}$

https://youtu.be/iL3mkIKo95M

Example 5

Find and simplify the derivative.

$f(x) = x^2\sqrt{1 - x^2}$

https://youtu.be/nDYAL3gX5Bs

Example 6

Find and simplify the derivative.

$$y = \frac{x}{\sqrt[3]{x^2+2}}$$

https://youtu.be/nnGL2gIq1mM

Example 7

Find and simplify the derivative.

$$f(x) = \left(\frac{2x-1}{x^2+2}\right)^2$$

https://youtu.be/fVAoLUvTzhA

Trigonometric Functions Chain Rule

For the following, let u be a differentiable function of x.

$$\frac{d}{dx}\sin u = u' \cdot \cos u \qquad \frac{d}{dx}\cos u = -\,u' \cdot \sin u$$

$$\frac{d}{dx}\tan u = u' \cdot \sec^2 u \qquad \frac{d}{dx}\cot u = -\,u' \cdot \csc^2 u$$

$$\frac{d}{dx}\sec u = u' \cdot \sec u \tan u \qquad \frac{d}{dx}\csc u = -\,u' \cdot \csc u \;\; cotu$$

Example 8

Find and simplify the derivative.

1. $y = \cos 2x$
2. $f(x) = \sin(x-1)$
3. $y = \cot 3x$

https://youtu.be/N5ipXH4iKXQ

Extension 1

Find and simplify the derivative. Be careful with parenthesis

1. $y = \sin 3x^2$
2. $y = (\sin 3)x^2$
3. $y = \sin(3x)^2$
4. $y = \sin^2 3x$

Sometimes we need to apply the chains rule multiple times…

Example 9

Find and simplify the derivative.

$f(x) = \cos^3 4t$

https://youtu.be/EtXwKSj_mx0

Example 10

Find the equation of the tangent line to the graph of $y = 2\sin x + \cos 2x$ at the point $(\pi, 1)$. Then determine all values of x within the open interval $(0, 2\pi)$ where the graph has horizontal tangents.

https://youtu.be/Zm4qEWuRAAM

Extension 2

Create a chart that organizes all Differentiation Rules covered in this chapter so far.