Unit 2: The Derivative

Section 2.1: Understanding the Basic Derivative

Objectives:

- Use limits to find the derivative of a function
- Find the slope of a tangent line to a curve at a point

There are two fundamental operations in Calculus: 1) Differentiation and 2) Integration.

Definition: The Derivative

The **derivative** of $f(x)$ with respect to x is the function $f'(x)$ where

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The derivative is a function of x. $f'(x)$ is commonly read as "*f prime of x".* A function is differentiable at x if its derivative exists at x. A function is differentiable on an open interval (a, b) if it is differentiable at every point in the interval.

Common Notation of Derivatives

$$f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx}f(x)$$

Example 1

Find the derivative using limits: $f(x) = x^3 - 2x$

https://youtu.be/53eynLXVDsU

A **Tangent Line** is a line that touches a graph at only one point. The slope of the tangent line (m_t) of a function $f(x)$ when $x = c$ is $f'(x)$ evaluated at c.

$$m_t = f'(c) = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

Example 2

Find the derivative, then find the slope at the points $(3,2)$, $(0,1)$ and $(-1,0)$ if possible. If not possible, explain why not.

$$f(x) = \sqrt{x+1}$$

https://youtu.be/mssWMYxe--A

Mathematics is the language for many other sciences and fields of study. Some of those studies use descriptive variables, that is variables that represent things like t for time, V for volume, or A for area, and so on. We can still find derivatives of functions that do not have x as the independent variable.

Example 3

1. Find $f'(x)$ given $f(x) = 2x^2 - 3$
2. Find the slope of the tangent line of $f(x)$ at the point (2, 5)
3. Find the equation of the tangent line at the point (2, 5)

https://youtu.be/1MymM_zV4zI

Example 4

1. Find the derivative for the function $y = \dfrac{3}{t}$

2. Find the slope of the tangent line when $t = 3$

3. Find the equation of the tangent line at $t = 3$

https://youtu.be/By_hly0nOqM

Theorem: Differentiability Implies Continuity

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Alternative Limit Form of the Derivative

The derivative of f at c is given by

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

This alternative form requires that the following one-sided limits exist and equal to each other.

$$f'(c) = \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} \qquad \text{and} \qquad f'(c) = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

Example 5

Show that the function $f(x) = |x - 3|$ is continuous at $x = 3$, but that it is not differentiable at $x = 3$.

https://youtu.be/1re6ticpW1A

Example 6

Show that the function $f(x) = x^{\frac{1}{3}}$ is continuous at $x = 0$, but that it is not differentiable at $x = 0$.

https://youtu.be/-zw1zGOTKpw

From the previous two examples we can conclude that a function f can be continuous at $x = c$ yet not be differentiable at c. On the contrary, if f IS differentiable at $x = c$, then f is continuous at c.

Differentiability Implies Continuity

If f IS differentiable at $x = c$, then f is continuous at $x = c$.

Extension 1

Summarize the relationship between continuity and differentiability.k