Unit 1: Limits

1.1 Finding Limits

Objectives:

- Estimate a limit using informal numerical and/or graphical approach
- Understand when limits fail to exist

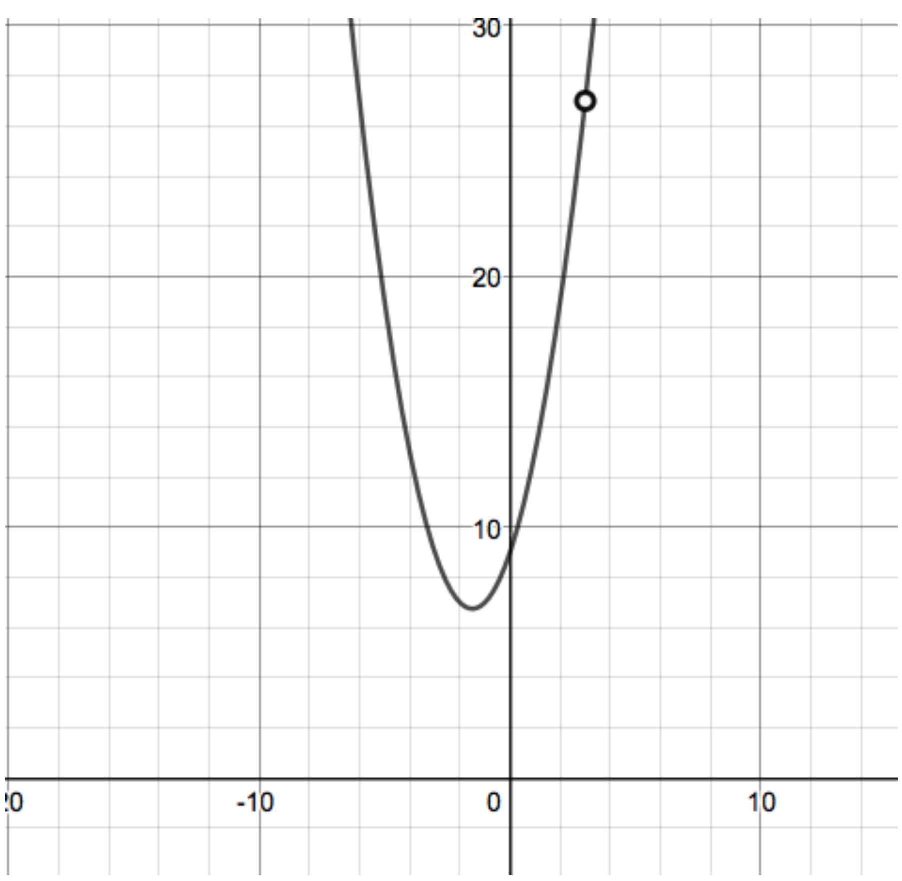
Introduction to Limits

Suppose we needed to graph the function $f(x) = \dfrac{x^3 - 27}{x - 3}$

Using rational function methods, we find the following characteristics fo the graph:

Y-intercept at the point $(0,9)$ and no X-intercept.

No vertical asymptote, yet a hole at the point where $x = 3$.



Graphed, we can see that no point exists when $x = 3$, yet how can we figure out what the y-value is of this undefined point?

Even though the graph does not exist at the point where $x = 3$, it does it exist very close to when $x = 3$. We can use the graph to give us "context clues", but those clues have to be consistent as seen in the following explanation.

The previous approach is the informal graphical approach. We can see a numerical approach by using the equation of the function rather than the graph. The idea here is to substitute numbers in for x closer and closer to when $x = 3$. We do this for both sides of $x = 3$ just as in the graphing approach. Consider the following table,

x	2.9	2.99	2.999	3	3.0001	3.001	3.01
f(x)				Undefined			

How might we find the corresponding of $f(x)$ for each x? By substituting each value of x into the original equation, $f(x) = \dfrac{x^3 - 27}{x - 3}$.

x	2.9	2.99	2.999	3	3.0001	3.001	3.01
f(x)	26.1100	26.9101	26.9910	Undefined	27.0009	27.0090	27.0901

Inspecting the trend of $f(x)$ values from the left and the right of $x = 3$ lead us to believe that the value of the undefined point when $x = 3$ is 27.

Informal Definition of a Limit

If $f(x)$ become arbitrarily close to a single number *L* as x approaches a value c from both sides, the limit of $f(x)$, as x approaches c, is *L.* The limit is written:

$$\lim_{x \to c} f(x) = L$$

Understand that existence of the function/graph is not a necessity for the limit to exist.

Example 1

Find the limit of $f(x)$ as $x \to 0$ given $f(x) = \begin{cases} -x + 2 & x < 0 \\ 10 & x = 0 \\ x + 2 & x > 0 \end{cases}$

Worked out solution: https://youtu.be/8JOy-QOIFUk

Example 2

Find the limit of $f(x)$ as $x \to 0$ given $f(x) = \dfrac{x}{\sqrt{x+1} - 1}$

Worked out solution: https://youtu.be/hvKHtbz-PIg

Not all limits exist.

Behavior that Indicates Nonexistence of a Limit

1. $f(x)$ approaches different values from the left and right side as $x \to c$
2. $f(x) \to \pm\infty$ as $x \to c$
3. $f(x)$ oscillates between two fixed values as $x \to c$.

Example 3

Show that $\lim_{x \to 0} \dfrac{|x|}{x}$ does not exist.

Worked out solution: https://youtu.be/zy-CvRknrYw

Example 4

Find $\lim_{x \to 0} \dfrac{1}{x^2}$ if possible. If not, explain why not.

Worked out solution: https://youtu.be/kKZAsvdtdO8

Example 5

Find $\lim_{x \to 0} \sin \dfrac{1}{x}$ if possible. If not, explain why not.

Worked out solution: https://youtu.be/9tHqhOBxdtw