Chapter 2 Test Part 2 Review
Chain, Implicit, and Related Rates

Section 1: Chain Rule Find the derivative.

1. $y = (7x+3)^4$

$y' = 28(7x+3)^3$

2. $y = 5\cos(9x+1)$

$y' = -45\sin(9x+1)$

3. $y = x(6x+1)^5$
$y' = (6x+1)^4(36x+1)$

4. $f(x) = \dfrac{3x}{\sqrt{x^2+1}}$ $\quad y' = \dfrac{3}{(x^2+1)^{\frac{3}{2}}}$

5. $s(t) = (t^2-1)^{\frac{5}{2}}(t^3+5)$
$y' = (t^2-1)^{\frac{3}{2}}(3t^4+5t^3-3t^2+25)$

6. $h(x) = \left(\dfrac{x+5}{x^2+3}\right)^2$
$y' = \dfrac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}$

Section 2: Implicit Differentiation Find the derivative with respect to x.

7. $x^2 + t^2 = 81$

$\dfrac{dt}{dx} = \dfrac{-x}{t}$

8. $x^4y^2 - 4 = xy^3$

$\dfrac{dy}{dx} = \dfrac{4x^3y - y^2}{3xy - 2x^4}$

9. $x^2\sin y = y\cos(2x^3)$

$\dfrac{dy}{dx} = \dfrac{-6x^2y\sin(2x^3) - 2x\sin y}{x^2\cos y - \cos(2x^3)}$

Section 3: Related Rates

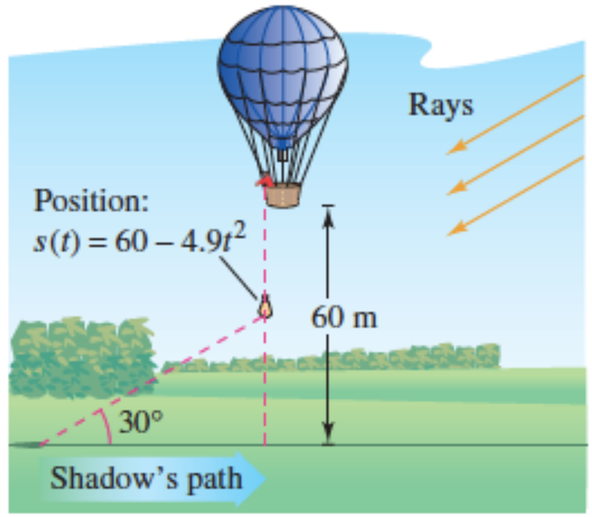
10. All edges of a cube are expanding at a rate of 8 cm per second. How fast is the surface area changing when each edge is 6.5 cm?
$624\,\dfrac{cm^2}{s}$

11. A sandbag is dropped from a hot air balloon at a height of 60 meters when the angle of elevation to the sun is 30^o. The position of the sandbag is given by the function:

$$s(t) = 60 - 4.9t^2$$

Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 35 meters.

$-38.34\ m/s$



12. An astronaut standing on the moon throws a rock upward. The height of the rock is given b the position function:

$$s(t) = -\frac{27}{10}t^2 + 27t + 6$$

Where *s* is measured in feet and *t* is in seconds.

a. Find the expressions for the velocity and acceleration the rock

$v(t) = \frac{-27}{5} + 27 \qquad a(t) = \frac{-27}{5}$

b. Find the time when the rock is at its highest point finding the time when velocity is zero. What is the height of the rock at this time? $t = 5,\ s(5) = 73.5$

c. How does the acceleration of the rock compare with the acceleration due to gravity on Earth?

Acceleration on moon is $\frac{-27}{5}\frac{ft}{s^2} = -5.4\frac{ft}{s^2}$ while Earth's is roughly $-32.17\frac{ft}{s^2}$

The negative symbolizes pull towards the celestial body.

Section 4: Find the equation of the tangent line.

13. $f(x) = \sqrt{1 - x^3}$ @ $(-2, 3)$ $y = -2(x+2) + 3$

14. $y = \frac{1}{2}\csc(2x)$ @ $\left(\frac{\pi}{4}, \frac{1}{2}\right)$ $y = 0\left(x - \frac{\pi}{4}\right) + \frac{1}{2}$

15. $y = \csc(3x) + \cot(3x)$ @ $\left(\frac{\pi}{6}, 1\right)$ $y = -3\left(x - \frac{\pi}{6}\right) + 1$

16. $x^2 + y^2 = 10$ @ $(3, 1)$ $y = -3(x - 3) + 1$

17. $x^2 - y^2 = 20$ @ $(6,4)$ $y = \frac{3}{2}(x - 6) + 4$

Section 5: Find the normal line to the graph at the given point. The *normal line* at a point is perpendicular to the tangent line at that point.

18. $x^2 + y^2 = 10$ @ $(3, 1)$ $y = \frac{1}{3}(x - 3) + 1$

19. $x^2 - y^2 = 20$ @ $(6,4)$ $y = -\frac{2}{3}(x - 6) + 4$

Section 6: Abstract Chain Rule

20. If $h(x) = f(g(x))$ find $h'(2)$ given that $g(2) = 3, g'(2) = -4, f'(2) = 7, f'(3) = 5$
$h'(x) = f'(g(x)) \cdot g'(x) = f'(g(2)) \cdot g'(2) = f'(3) \cdot (-4) = 5 \cdot (-4) = -20$

21. If $h(x) = f(g(x))$ find $h'(2)$ given that $g(x) = x^3 - 8$, and $f'(x) = \sqrt{3x^2 + 4}$
$g'(x) = 3x^2$ then $h'(x) = f'(g(x)) \cdot g'(x) = f'(g(2)) \cdot g'(2)$

Use the functions to find their values so: $g(2) = 0$, $g'(2) = 12$, $f'(0) = 4$
Now $f'(g(2)) \cdot g'(2) = f'(0) \cdot 12 = 4 \cdot 12 = 48$

22. If $h(x) = f(3x^2 + g(x^2))$ and $g(4) = -6, g'(4) = -2, f'(6) = -12$, and $f(6) = -3$
find $h'(2)$. $= f'(12 + g(4)) \cdot (12 + 4g'(4)) = -12 \cdot 4 = -48$

23. If $F(x) = f(1+2x) \cdot g(x)$ find $F'(1)$ given the table of values below. $F'(1) = -12$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-2	7	3	1
3	6	-3	2	4