6.8 Non-Linear Systems: Polynomials

Objectives:

- Use substitution and elimination methods to solve systems involving polynomials.

First, remember the concepts associated with polynomials:

The Rational Zero Test

If a rational number $\frac{r}{s}$ (written in lowest terms) is a zero of the polynomial function

$$f(x) = a_n x^n + \ldots + a_1 x + a_0$$

where the coefficients $a_n, \cdots, a_1$ are *integers* with $a_n \neq 0$ and $a_0 \neq 0$, then

- r is a factor of the constant term a_0 and
- s is a factor of the leading coefficient a_n.

Remainders and Factors

If the remainder is 0 when one polynomial is divided by another polynomial, the divisor and the quotient are factors of the dividend.

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Factor Theorem

A polynomial function $f(x)$ has a linear factor $x - a$ if and only if

$$f(a) = 0.$$

Zeros, x-Intercepts, Solutions, and Factors of Polynomials

Let $f(x)$ be a polynomial. If r is a real number that satisfies any of the following statements, then r satisfies all the statements.

- r is a zero of the function f
- r is an x-intercept of the graph of the function f
- $x = r$ is a solution, or root, of the equation $f(x) = 0$
- $x - r$ is a factor of the polynomial $f(x)$

There is a one-to-one correspondence between the linear factors of $f(x)$ that have real coefficients and the x-intercepts of the graph of f.

Number of Zeros

A polynomial of degree n has at most n distinct real zeros.

End Behavior of Polynomial Functions

When $|x|$ is large, the graph of a polynomial function closely resembles the graph of its highest degree term.

When a polynomial function has *odd* degree, one end of its graph shoots upward and the other end downward.

When a polynomial function has *even* degree, both ends of its graph shoot upward or both ends shoot downward.

Multiplicity

If $x - r$ is a factor of a polynomial that occurs m times in the complete factorization, then r is a zero of multiplicity m.

Example 1

Solve the system.

$$\begin{cases} y = -7x^2 - 7 \\ y = -x^3 - x \end{cases}$$

Example 2

Solve the system.

$$\begin{cases} y = -2x^3 - 2x^2 + 3 \\ y = x^3 + 5x - 3 \end{cases}$$

Example 3

Solve the system.

$$\begin{cases} y = -2x^3 - 13x^2 + 2x - 5 \\ y = -x^3 - 9x^2 - 9x + 1 \end{cases}$$

Example 4

Solve the system.

$$\begin{cases} y = x^4 + 3x^3 - 5x^2 - 2x + 3 \\ y = 3x^3 + 4x^2 + 2x - 9 \end{cases}$$