Chapter 6: Systems of Equations and Matrices

Objectives:

- Solve systems of equations by graphing, substitution, and elimination.
- Recognize consistent and inconsistent systems
- Solve applications using systems.

A system of equations is a set of two or more equations in two or more variables. Systems are identified by how many equations and variables it contains. A system that has 3 equations and 2 variables is identified as a 3 x 2 system, read three by two.

Linear systems are systems where the variables of each equation are to the power of 1. A non-linear system is one where an equation has a variable raised to a power other than 1.

$$\begin{aligned} 2x - 5y + 3z &= 1 \\ x + 2y - z &= 2 \\ 3x + y + 2z &= 11 \end{aligned} \qquad \begin{aligned} 2x + 5y + z + w &= 0 \\ 2y - 4z + 41w &= 5 \\ 3x + 7y + 5z - 8w &= -6 \end{aligned} \qquad \begin{aligned} x^2 + y^2 &= 25 \\ x^2 - y &= 7 \end{aligned}$$

A solution of a system of equations is a set of values that satisfy all the equations in the system. In the first system above, substituting $x = 1, y = 2$, and $z = 3$ gives:

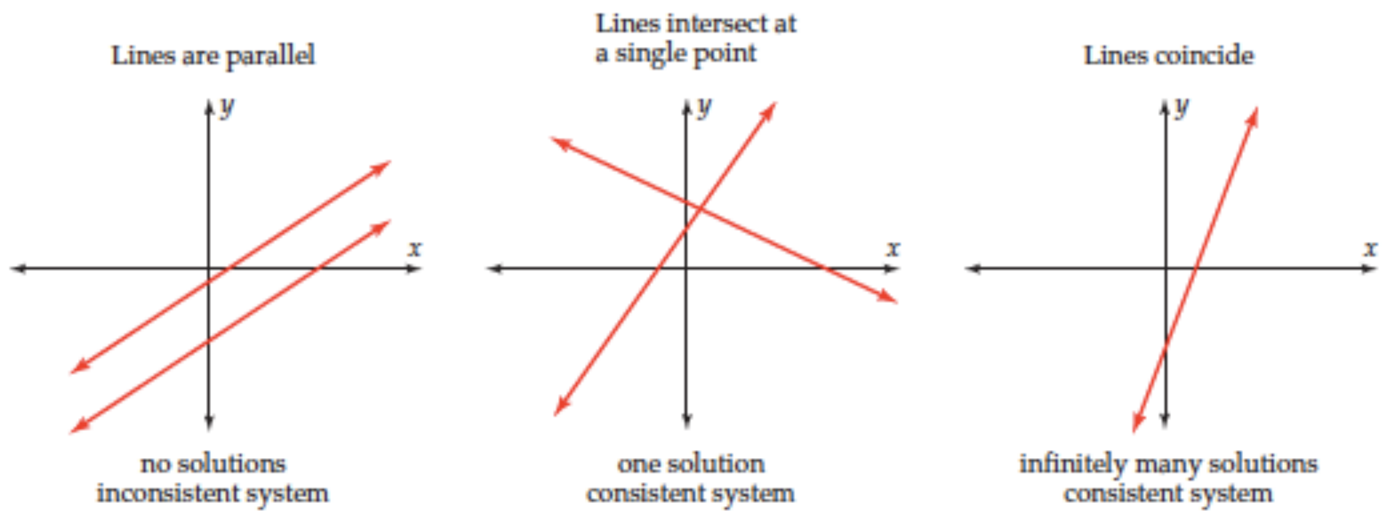
$$\begin{aligned} 2x - 5y + 3z &= 2(1) - 5(2) + 3(3) = 2 - 10 + 9 = 1 \\ x + 2y - z &= (1) + 2(2) - 3 = 1 + 4 - 3 = 2 \\ 3x + y + 2z &= 3(1) + (2) + 2(3) = 3 + 2 + 6 = 11 \end{aligned}$$

The set of values $x = 1, y = 2$, and $z = 3$ makes all equations in the system true, therefore is a solution to the system. A solution to a system is a point where all the graphs of the equations intersect. For now we will consider Linear Systems of equations.

Linear Systems and Number of Solutions

The graphs in linear systems with two variables are lines and so there are exactly three geometric possibilities.

- The lines may be parallel and have no point of intersection
- The lines may intersect at a single point.
- The lines may coincide, that is, the lines may be graphed one on top of the other.



A consistent system is a system that has a solution, at least one intersection point or point in common. An inconsistent system is a system that has no solution, no point in common.

Solving Systems Algebraically

Though solving by graphing is a valid method, we will explore algebraic methods to produce exact solutions. These algebraic methods include **substitution** and **elimination.**

Example 1: Solving a system using Substitution

$$\begin{cases} 3x - y = 12 \\ 2x + 3y = 2 \end{cases}$$

Example 2: Solving a system using Elimination

$$\begin{cases} x - 3y = 4 \\ 2x + y = 1 \end{cases}$$

Example 3: An inconsistent system

$$\begin{cases} 2x - 3y = 5 \\ 4x - 6y = 1 \end{cases}$$

Example 4: Infinitely many solutions

$$\begin{cases} 2x - 4y = 6 \\ -3x + 6y = -9 \end{cases}$$

Using Parameters to Write Solutions

It is by convention that solutions to consistent systems that have infinitely many solutions be represented by a variable called a parameter. In Example 4, let $y = t$ and substitute this value into one of the equations.

$$\begin{aligned} 2x - 4t &= 6 \\ x &= 3 + 2t \end{aligned}$$

The solutions can be written as $x = 3 + 2t$ and $y = t$. Individual numerical solutions can be found by substituting in real values for t. Here are some cases fo values of t:

$$\begin{array}{ll} t = 1 & x = 5, y = 1 \\ t = -2 & x = -1, y = -2 \\ t = 0 & x = 3, y = 0 \end{array}$$

Larger Linear Systems of Equations

Elimination can be used to solve large systems of equations. Equations are combined into pairs with one equation common in all pairs. These pairs are reduced and solve using elimination then substituted back to find remaining values or variables.

Example 5

$$\begin{cases} 2x + y - z = -1 \\ -x - 3y + z = 5 \\ x + 4y - 2z = -10 \end{cases}$$

Applications of Systems

Systems of equations model many real world situations. The simplest ones involve two quantities and two linear relationships between these quantities.

Example 6

A local high school fine arts program is having its fall program. 575 people attend and ticket sales total $2,575. If tickets cost $5 for adults and $3 for children, how many adults and how many children attended the program?

Example 7

A cafe sells two kinds of coffee in bulk. The Mexican coffee sell for $7.00 per pound and the Guatemalan coffee sells for $4.50 per pound. The owner of the cafe wishes to sell a blend that would sell for $5.00 per pound. How much of each coffee type should be used in the blend?