Section 4.2 Real Zeros of Polynomials

Objectives:

- Find all rational zeros of a polynomial function
- Use the Factor Theorem to identify factors of a polynomial
- Factor a polynomial completely
- Find Upper and Lower Bounds of polynomial zeros

When a polynomial has integer coefficients, all of its rational zeros can be found by using the Rational Zero Test.

Rational Zero Test

If a rational number $\frac{r}{s}$ is a zero of a polynomial function $p(x)$, then

- r is a factor of the constant term a_0
- s is a factor of the leading coefficient a_n

Conditions for Rational Zeros Test

1. Can only be used on polynomials that have a nonzero constant term.
2. Polynomials must only have integer coefficients.
3. Applies only to polynomials with a positive leading coefficient.

By finding all numbers that satisfy these condition, a list of possible rational zeros can be determined. The polynomial must be evaluated at each number in the list to verify that it is indeed a zero of $p(x)$.

Example 1

For the polynomial function $p(x) = 2x^4 + x^3 - 17x^2 - 4x + 6$

a) Find all possible rational zeros of $p(x)$
b) Find the rational zeros of $p(x)$

https://youtu.be/VyctB8s-MHU

Example 2

For the polynomial function $p(x) = \frac{2}{3}x^4 + \frac{1}{2}x^3 - \frac{5}{4}x^2 - x - \frac{1}{6}$

a) Find all possible rational zeros of $p(x)$
b) Find the rational zeros of $p(x)$

https://youtu.be/8_4O4XbMaJA

Once the zeros of a polynomial have been found the Factor Theorem can be used to factor the polynomial. This can lead to the discovery of additional zeros.

Example 3
Use the result from Example 1 to find all zeros of the polynomial
$p(x) = 2x^4 + x^3 - 17x^2 - 4x + 6$

https://youtu.be/iCuh3a3Z6ck

Example 4
Completely Factor: $p(x) = 2x^5 - 10x^4 + 7x^3 + 13x^2 + 3x + 9$
Thus, write $p(x)$ in factored form.

https://youtu.be/mS7HCZoh6MQ

Upper and Lower Bounds of Polynomial Zeros

Let $p(x)$ be a polynomial that has zeros. Also, m and n are real numbers such that $m < n$. If all the zeros of $p(x)$ lie between m and n, then m is a Lower Bound and n is an Upper Bound for the zeros of $p(x)$. If we can determine where these bounds are located, we can lessen the amount of possible rational zeros we have to test.

Bounds Test

Let $p(x)$ be a polynomial with a positive leading coefficient be divided by $x - d$.

- If d is a positive number and every number of the quotient and remainder of the synthetic division is nonnegative, then d is an upper bound for the real zeros of $p(x)$
- If d is a negative number and the numbers of the quotient and remainder alternate signs, (+) and (-), with 0 being considered as either, then d is a lower bound for the real zeros of $p(x)$.

Example 5
Find the upper and lower bounds of $f(x) = x^6 + x^3 - 7x^2 - 3x + 1$

https://youtu.be/WBiHEqcfpcA

Guide to Finding Zeros of Polynomials

1. Ue the Rational Zero Test to find all possible rational zeros of $p(x)$.
2. Determine the upper and lower bounds.
3. Use the factor theorem and remainder theorem to find all rational zeros.
4. Write $p(x)$ as the product of linear factors, one for each rational zero, and another factor.
5. If one of the factors has a degree of 2, find its zeros by factoring, completing the square, or by using the quadratic formula.

Example 6

Find the lower and upper bound for the zeros. Find all zeros of $p(x)$. Then completely factor $p(x)$ and write in factored form.

$$p(x) = 2x^5 - 7x^4 - 3x^3 + 17x^2 + 5x - 6$$

https://youtu.be/dwNlhOtxjNg